$C=KD$, where $K$ is a positive fraction in its lowest terms or a positive integer, and $D$ is a positive integer as above determined.

The second condition is not independent of the first, because, if $K'$ is substituted for $2K+1$ and $2D'$ for $D$, in the above set of relations, then we have

$$\frac{B+C}{B-C}=\frac{D'K'+D+'D'K'-D'}{D'K'+D'-D'K'+D'}=\frac{2D'K'}{2D'}=K',$$

which is the second condition. Hence the set of values found above includes all the possible relations which make $A$, $B$, and $C$ positive integers. The values of $A$ and $B$ may be interchanged, and $A$, $B$, and $C$ may each be multiplied by a common factor without changing the value of the original ratio.

Also solved by A. H. Holmes.

184. Proposed by E. B. ESCOTT, University of Michigan, Ann Arbor, Mich.

Prove that $\frac{\pi}{12}=\tan^{-1}\frac{1}{2^2}+\tan^{-1}\frac{1}{8^2}+\tan^{-1}\frac{1}{30^2}+\tan^{-1}\frac{1}{112^2}+\ldots$, where 2, 8, 30, 112..., is a recurring series with the recursion formula $u_n=4u_{n-1}-u_{n-2}$.

Solution by the PROPOSER.

If we reduce $\sqrt{3}$ to a continued fraction, we get for the convergents,

$$\frac{1}{1},\ \frac{2}{1},\ \frac{5}{3},\ \frac{7}{4},\ \frac{19}{11},\ \frac{26}{15},\ \frac{71}{41},\ \frac{97}{56},\ \frac{265}{153},\ \frac{362}{209},\ \ldots$$

The alternate convergents,

$$\frac{1}{1},\ \frac{5}{3},\ \frac{19}{11},\ \frac{71}{41},\ \frac{265}{153},\ \ldots$$

are formed by taking the ratios of the corresponding terms of the two recurring series

$$\begin{array}{l} 1,\ 5,\ 19,\ 71,\ 265,\ \ldots \\ 1,\ 3,\ 11,\ 41,\ 153,\ \ldots \end{array}$$

both having the same scale of relation $u_n=4u_{n-1}-u_{n-2}$.

We find by the usual methods that the $n$th terms of the two series are

$$\frac{\alpha^n-\beta^n}{\alpha-\beta}+\frac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta}=u_n+u_{n-1},\ \text{and}\ \frac{\alpha^n-\beta^n}{\alpha-\beta}-\frac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta}=u_n-u_{n-1},$$

where $\alpha$ and $\beta$ are the roots of the equation $x^2-4x+1=0$.

Taking the differences of the $\tan^{-1}$ function of the reciprocals of the convergents, we have

$$\tan^{-1}\frac{1}{1}-\tan^{-1}\frac{3}{5}=\tan^{-1}\frac{1}{4}$$

$$\tan^{-1}\frac{3}{5}-\tan^{-1}\frac{11}{19}=\tan^{-1}\frac{1}{64}$$

$$\tan^{-1}\frac{11}{19}-\tan^{-1}\frac{41}{71}=\tan^{-1}\frac{1}{900}$$

$$\tan^{-1}\frac{41}{71}-\tan^{-1}\frac{153}{265}=\tan^{-1}\frac{1}{12544}$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$\tan^{-1}\frac{u_n-u_{n-1}}{n_n+u_{n-1}}-\tan^{-1}\frac{u_{n+1}-u_n}{u_{n+1}+u_n}=\tan^{-1}\frac{1}{4u_n{}^2}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots$$

Adding, we have

$$\tan^{-1}\frac{1}{1}-\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{4}-\frac{\pi}{6}=\frac{\pi}{12}=\tan^{-1}\frac{1}{2^2}+\tan^{-1}\frac{1}{8^2}+\tan^{-1}\frac{1}{30^2}+\ldots$$

$$+\tan^{-1}\frac{1}{(2u_n)^2}+\ldots$$

## PROBLEMS FOR SOLUTION.

### ALGEBRA.

360. **Proposed by CHARLES C. GROVE, Columbia University, New York.**

A bridge club of 28 members has 27 meetings. There are 7 tables with 4 members at each table. Can the players be so arranged that at the end of the season (27 meetings) each member will have played *with* every other member one game and *against* every other member two games, one game meaning one meeting; and how?

361. **Proposed by C. E. GITHENS, Ph. D., Wheeling, W. Va.**

Find three integral values for $[-10+9\sqrt{(-3)}]^{1/3}+[-10-9\sqrt{(-3)}]^{1/3}$. A solution not involving a cubic is desired.